

Closed book/closed notes. Show all of your work clearly in the space provided or on the additional page at the end of the exam. If the additional page is used, be sure to clearly label the content for each problem. Be sure to *read each problem carefully*. You should answer all 5 questions, and if time permits, you may wish to answer the bonus question as well. Note that the exam is double sided.

1. (20 points)  $f(n)$  describes the actual time consumed by an algorithm on an input of size  $n$ . Give the corresponding big-oh notation for  $f(n)$  in each part. (In order to receive partial credit, you must explain your reasoning.)

(a)

$$f(n) = n + \log(n^2) \quad \text{for all } n > 0$$

(b)

$$f(n) = \begin{cases} 2^n + 10^8 & 0 < n \leq 10^8 \\ 4n^3 + 2n(n - 2n^2) - 10^8 & n > 10^8 \end{cases}$$

(c)

$$f(n) = n^2 + n^5 \quad \text{for all } n > 0$$

(d)

$$f(n) = \begin{cases} 3n + 2^n + 4 & 0 < n \leq 4 \\ \sqrt{n} + n^2\sqrt{n} & 4 < n \leq 16 \\ 1^n & n > 16 \end{cases}$$



2. (10 points) What causes the compiler to produce the C4786 warning message?

3. (15 points) Compare and contrast the `set` and `map` data structures.



4. In the first laboratory assignment, you were to create a simple spell checking program. Assume that the dictionary contains  $n$  words and that the document to be checked contains  $m$  words.
- (a) (15 points) Using big-oh notation, describe the overall worst case time complexity for your program. Be sure to explain your reasoning.

(b) (23 points) Suppose that the words in the dictionary are in alphabetical order and that your program is modified so that:

1. all of the words in the document are first sorted, (assume this can be done in  $O(\log m)$  time)
2. `prevPos` is set to the beginning of the dictionary list
3. the alphabetically first word in the document is searched for in the dictionary beginning at `prevPos`
4. (a) if the word was found, we set `prevPos` to the position in the dictionary where the word was found  
(b) if the word wasn't found, we add it to our list of "misspelled" words and leave `prevPos` unchanged
5. the next word in the document is then searched for in the dictionary beginning at `prevPos`
6. loop back to step 4 until the end of the document is reached

Using big-oh notation, describe the overall worst case time complexity for this modified program? Be sure to explain your reasoning.



5. (17 points) Prove the following relation using mathematical induction:

$$\sum_{k=0}^n x^k = \frac{x^{n+1} - 1}{x - 1} \quad x \neq 1$$



**bonus** (5 points) Write an  $O(1)$  algorithm to compute the following summation:

$$\sum_{k=0}^n x^k$$

Note: No partial credit will be awarded for the bonus problem.



Additional work area for any problem. Clearly identify to which problem the work on this page is related.



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