



[**You may use one side of an 8.5 × 11 inch sheet of paper, no calculator**] Show all of your work clearly in the space provided or on the additional page at the end of the exam. If the additional page is used, clearly identify to which exam question it is related. Be sure to **read each problem carefully**. You should answer all 4 questions. Note that the exam is double sided.

1. (15 points) Find $\sum_{k=-2}^{200} (2k+2)$. Be sure to note the bounds on the summation and show all your work. *Remember: you may not use a calculator.*



2. (20 points) What is the maximum number of times during the execution of quicksort that the largest element can be moved, for an array of N elements? Be sure to justify your answer.



3. (25 points) Prove by induction that for $n > 4$, $2^n > n^2$.

4. Given a set of n numbers, we wish to find the i largest in sorted order using a comparison-based algorithm. Analyze the running times of the algorithms described below in terms of n and i . Justify your answers.

(a) (10 points) Sort the numbers using Mergesort and list the i largest.

(b) (15 points) Find the largest of the first n elements, swap it with the last element, decrement n by one and repeat until $n = i$.

(c) (15 points) Select a random ordering for the elements. Find the largest of the first $n - i$ elements. If it is smaller than all of the remaining elements, list it and the remaining elements and stop. If not, select another unique order (i.e., an order that has not been encountered yet) and repeat.

Assume that a unique random ordering can be produced in linear time.



Additional work area for any problem. Clearly identify to which problem the work on this page is related.



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