[**You may use one 8.5 $\times$ 11 inch sheet of paper and a calculator.**] Show all of your work clearly in the space provided or on the additional page at the end of the exam. If the additional page is used, clearly identify to which exam question it is related. Be sure to **read each problem carefully**. You should answer all 4 questions. Note that the exam is double sided.

1. (10 points) Both Bézier and B-Spline curves are written in terms of a parameter u:

$$x = x(u) \qquad\qquad y = y(u)$$

Explain the benefits of such a parameterization.

2. (10 points) Explain what is meant by **extrapolation** in the context of solving ordinary differential equations.

3. (15 points) Given the following second order differential equation:

$$\frac{1}{2}\frac{d^2}{dt^2}x + 6\frac{d}{dt}x + 50x = 24\sin(10t)$$

where, at $t = 0$, $x = 0$ and $\frac{d}{dt}x = 0$. Write the corresponding set of first order differential equations.

4. Consider the function $f(x)$ described by the data set given in part **(a)**.

(a) (15 points) Fill in the divided difference table for $f(x)$. Show your work for partial credit.

x_i	$f(x_i)$	
1	4.5	
2	18.0	
4	129.0	
5	250.5	
7	685.5	

(b) (10 points) What order polynomial is required to pass through all the points? Why?

(c) (14 points) Write the second order polynomial that passes through the first three points.

(d) (14 points) Use a second order polynomial approximation and numerical methods to determine the derivative of $f(5.2)$.

5. (12 points) Suppose we wish to find $\int_1^{11} f(x)dx$ for some $f(x)$ defined on $x \in [1, 11]$. Rank the following methods in terms of accuracy. Be sure to justify your rankings.

a zeroth-order fit with $h = 0.2$
b $\frac{3}{8}$ Simpson's rule with $h = 0.2$
c Monte Carlo method with $N = 6$
d $\frac{1}{3}$ Simpson's rule with $h = 0.2$

Additional work area for any problem. Clearly identify to which problem the work on this page is related.